177. Proposed by C. S. Venkataraman, Sree Kerala Varma College, Trichur, South India.

If s is the semi-perimeter and R, r, r_1, r_2, and r_3 are the circum-, in-, and ex-radii, respectively, of a triangle, prove that

$$\frac{R}{r^2} \geq \frac{2s^2}{r_1 r_2 r_3} \; .$$

Solution by Stanley Rabinowitz, Polytechnic Institute of Brooklyn.

We start with the known inequality, $R \geq 2r$, with equality if and only if the triangle is equilateral. It is also known that $r r_1 r_2 r_3 = K^2$ where K is the area of the triangle (see N. A. Court, College Geometry, p. 79). Since also $K = rs$, we have $r r_1 r_2 r_3 = r^2 s^2$. Finally,

$$\frac{R}{r^2} \geq \frac{2}{r} = \frac{2s^2}{r_1 r_2 r_3} \; .$$